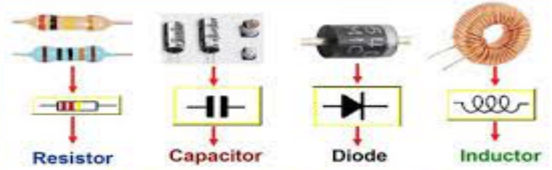
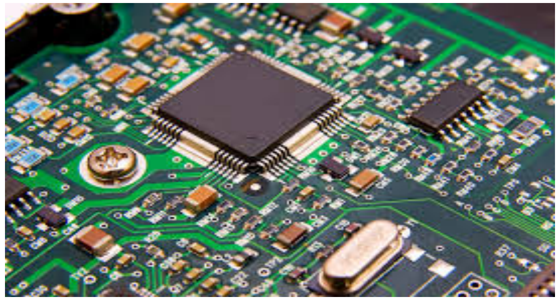
Electronics 1

BSC 113

Fall 2022-2023

Lecture 4





Techniques of Circuit Analysis

INSTRUCTOR

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➢ Contents

Kirchhoff's law

- Current
- Voltage

❑ 2.1 Kirchhoff's current law

- In Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.

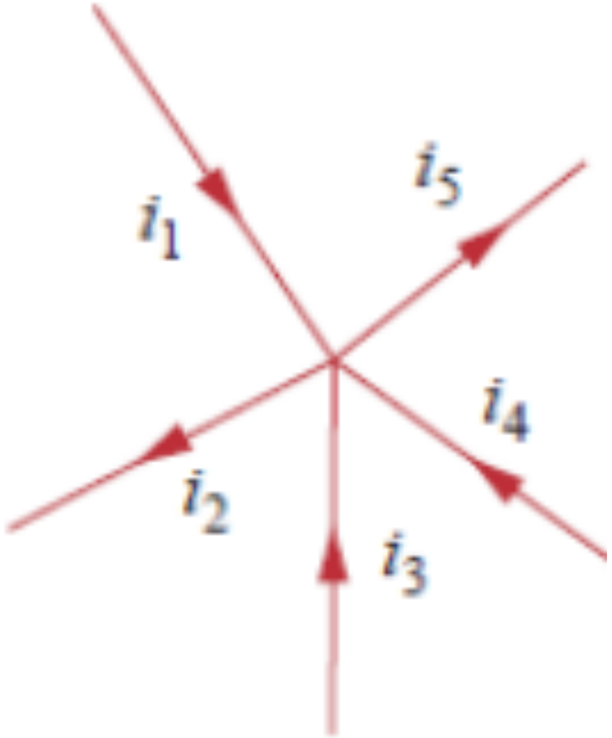
- Mathematically, KCL implies that

$$\sum_{n=1}^{N} i_n = 0$$

- where N is the number of branches connected to the node and i_n is the n-th current entering (or leaving) the node.

❑ 2.1 Kirchhoff's current law

- As shown in figure1, by this law, currents entering a node may be regarded as positive, while currents leaving the node may be taken as negative or vice versa.



$$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$$

Fig.1: Example on KCL

❑ 2.1 Kirchhoff's current law

- As shown in figure 2, the sum of the currents entering a node is equal to the sum of the currents leaving the node.

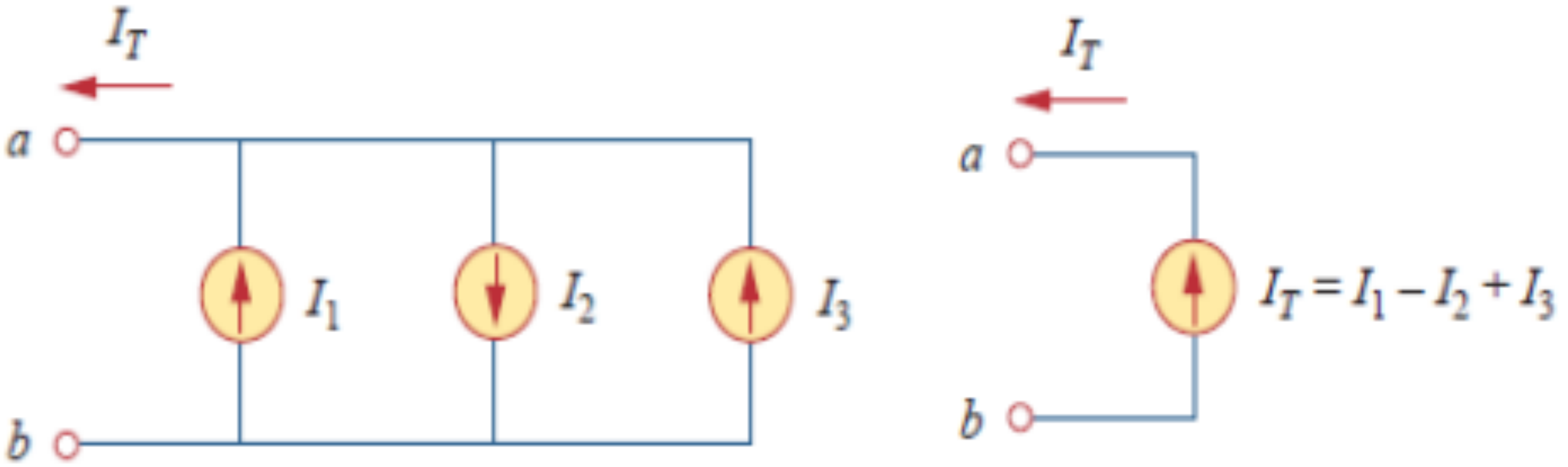


Fig.2: Example on KCL

❑ 2.2 Kirchhoff's voltage law

- In Kirchhoff’s voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.

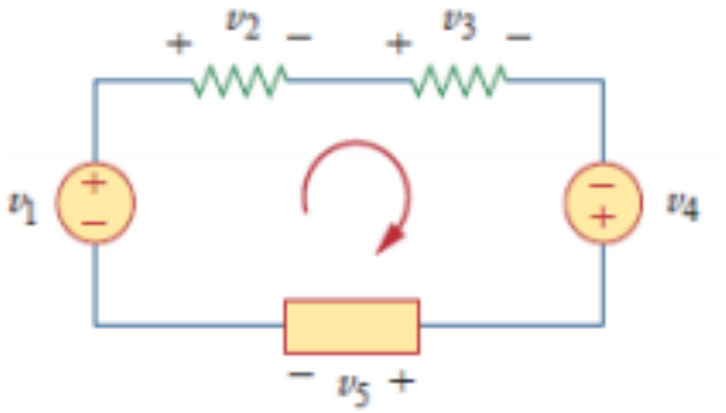
- Mathematically, KVL implies that

$$\sum_{m=1}^{M} v_m = 0$$

- where M is the number of voltages in the loop (or the number of branches in the loop) and v_m is the m-th voltage.

❑ 2.2 Kirchhoff's voltage law

- As shown in figure 3, by this law, The sign on each voltage is the polarity of the terminal encountered first as we travel around the loop. We can start with any branch and go around the loop either clockwise or counterclockwise.
- In other words, sum of voltage drops = sum of voltage rises



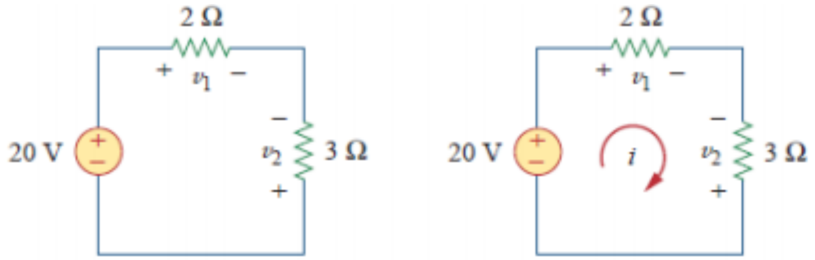
$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

$$v_2 + v_3 + v_5 = v_1 + v_4$$

Fig. 3: Example on KVL

❑ Example 2.1

- For the circuit in the following figure, find voltages v1 and v2.



- Answer: To find v1 and v2 we apply Ohm's law and Kirchhoff's voltage law. Assume that current i flows through the loop as shown in Fig. From Ohm's law,

$$v_1=2*i,\ v_2 = -3*i.$$

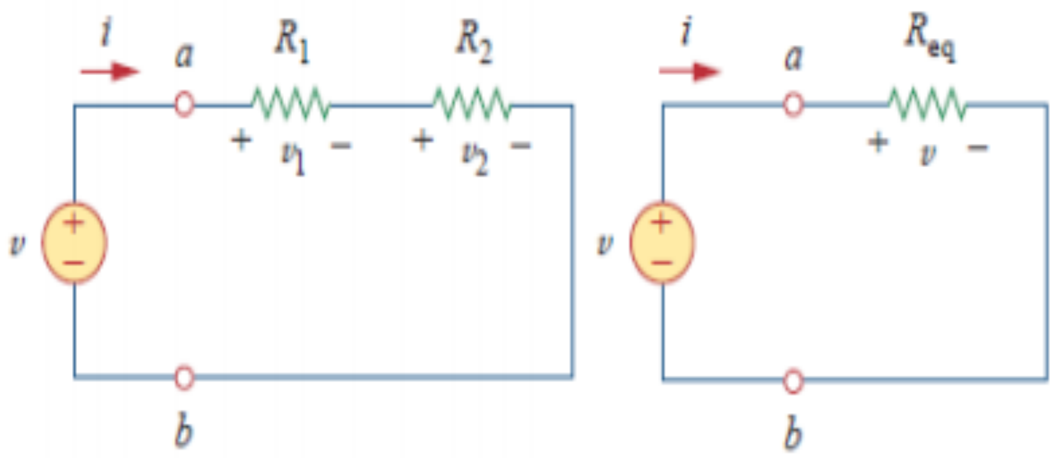
- Applying KVL around the loop gives

$$-20 + v_1 - v_2 = 0$$

we obtain i = 4 A. $v_1= 8$ V and $v_2 = -12$ V

❑ 2.3 Series and parallel resistance

- The process of combining the resistors is facilitated by combining two of them at a time. Consider the single-loop circuit of figure 4. The two resistors are in series, since the same current i flows in both. Applying Ohm's law to each of the resistors, we obtain



$$v_1 = i * R_1 \text{ and } v_2 = i * R_2$$

$$-v + v_1 + v_2 = 0$$

$$v = v_1 + v_2 = i(R_1 + R_2)$$

$$i = \frac{v}{(R_1 + R_2)}$$

$$v = i\, R_{eq} \qquad R_{eq} = R_1 + R_2$$

❑ 2.3 Series and parallel resistance

- Now we can say the equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.

$$R_{eq} = \sum_{i=1}^{N} R_i$$

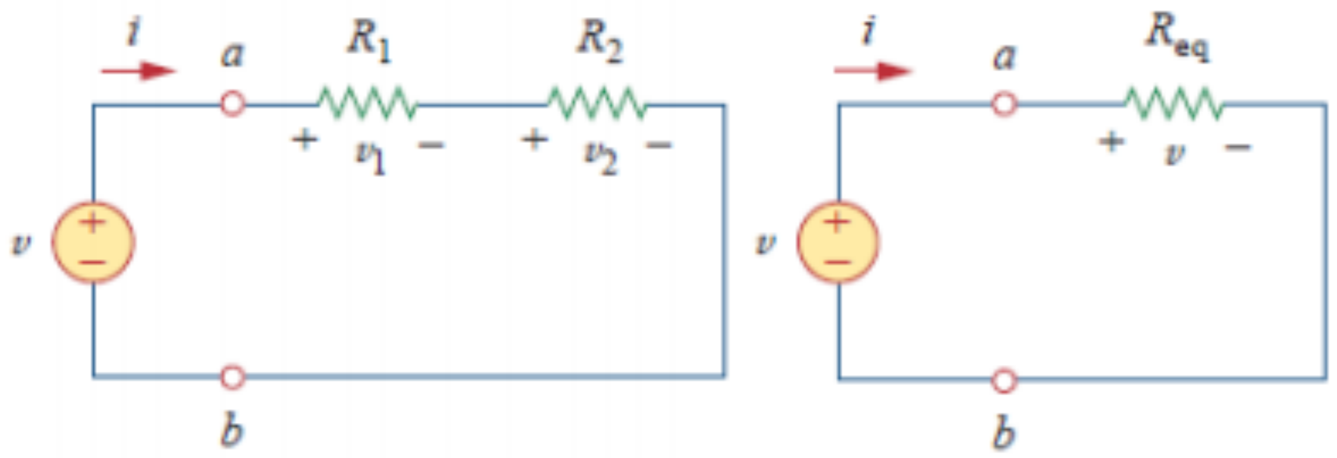


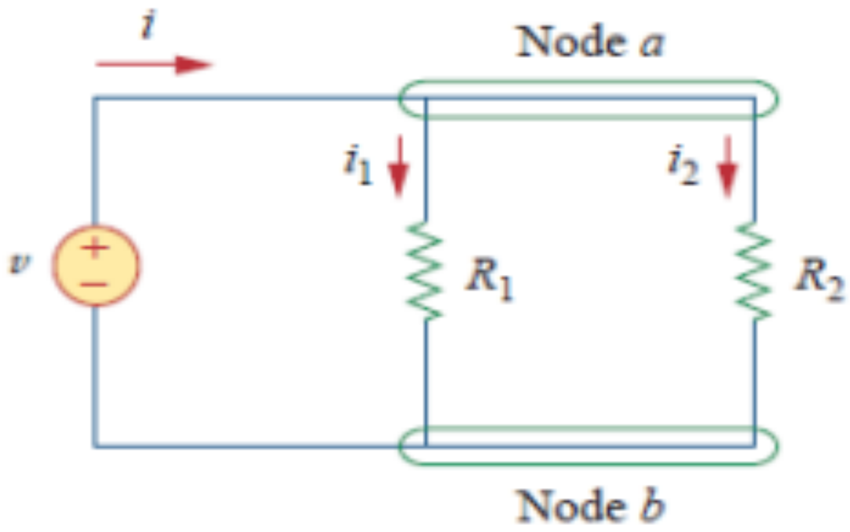
Fig. 4: A single-loop circuit with two resistors in series.

❑ 2.3 Series and parallel resistance

- If $R_1 = R_2 = \cdots = R*N = R$, then

$$R_{eq} = N*R$$

- where two resistors are connected in parallel and therefore have the same voltage across them as shown in figure 5. From Ohm's law,



$$v = i*R_1 = i*R_2$$

$$i_1 = \frac{v}{R_1}, \qquad i_2 = \frac{v}{R_2}$$

$$i = i_1 + i_2 = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v}{R_{eq}}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

❑ 2.3 Series and parallel resistance

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N}$$

- The equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum.

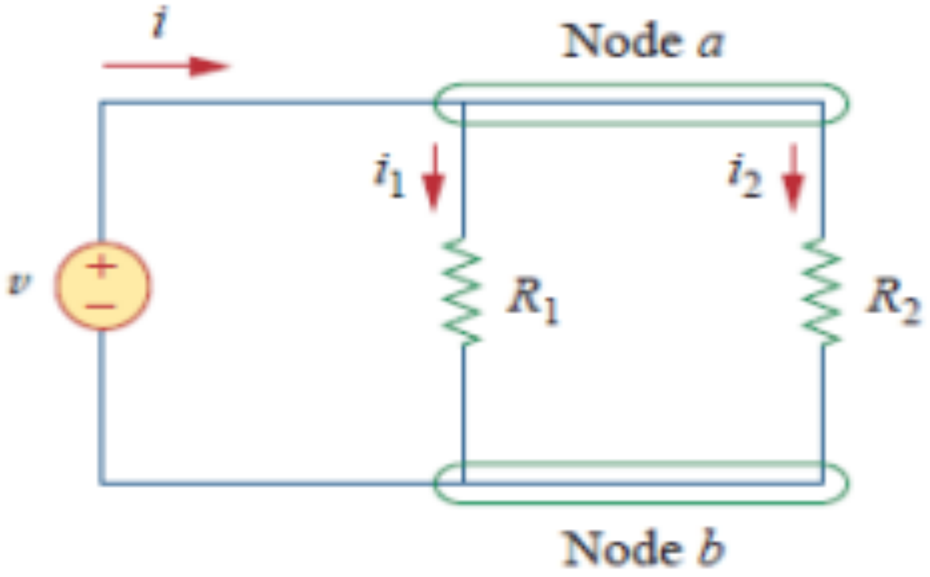


Fig. 5: A single-loop circuit with two resistors in parallel.

2.3 Series and parallel resistance

- Note that the equivalent resistance is always smaller than the resistance of the smallest resistor in the parallel combination. If

$$R_1 = R_2 = \cdots = R_N = R, \text{ then}$$

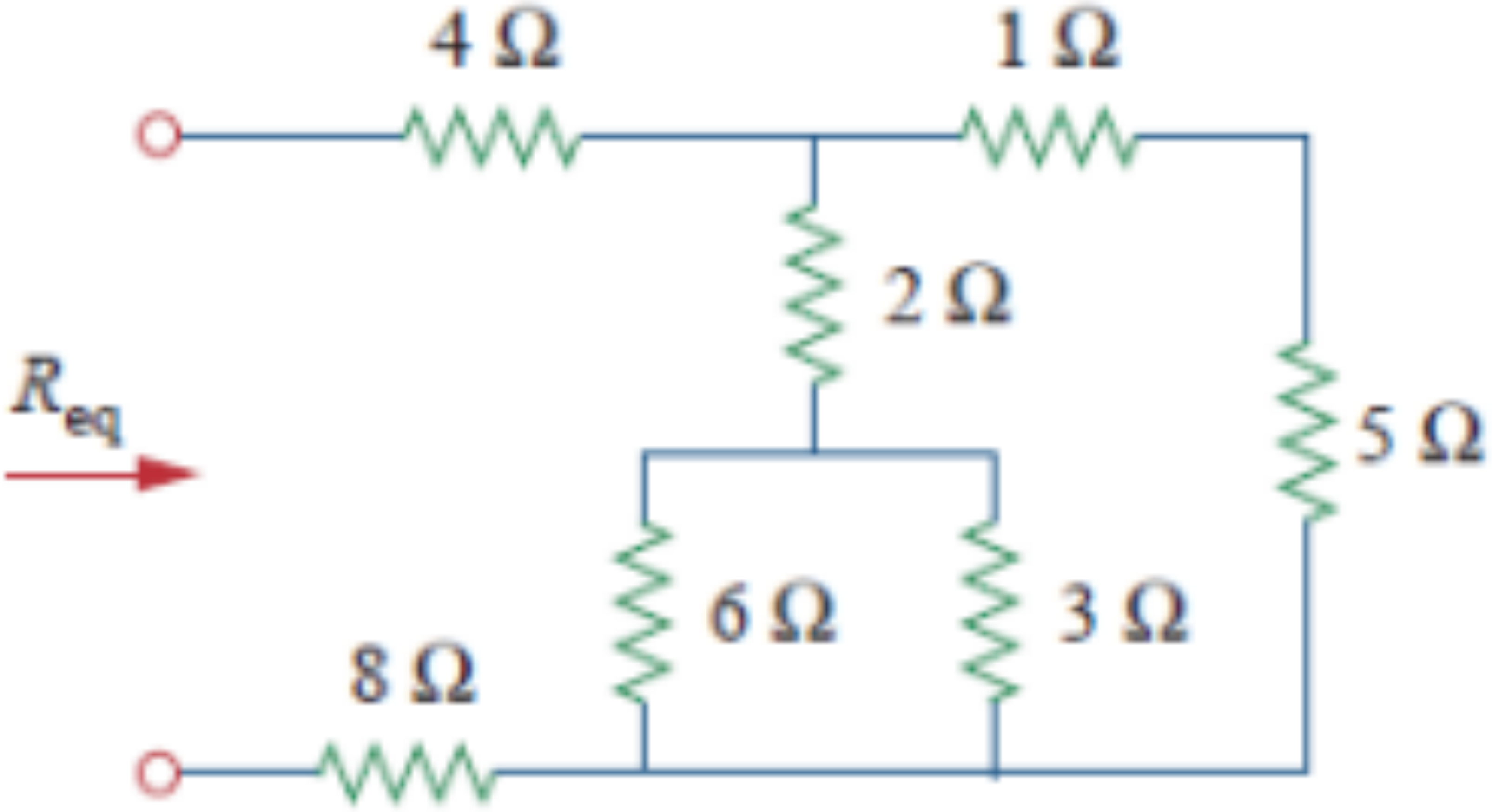
$$R_{eq} = \frac{R}{N}$$

- The equivalent conductance of resistors connected in parallel is the sum of their individual conductance.

$$G_{eq} = G_1 + G_2 + \cdots + G_N$$

❑ Example 2.2:

➢ Find Req for the circuit shown in the following figure



❑ Example 2.2:

Answer:

Two resistors 6 and 3 parallel

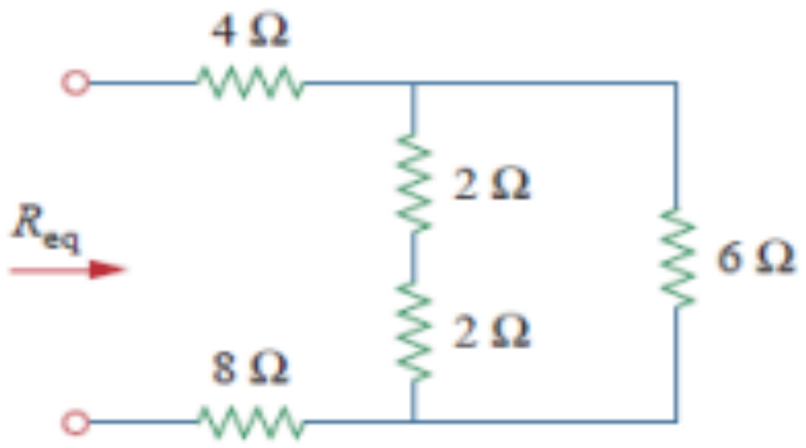
6 // 3 = 2 Ω

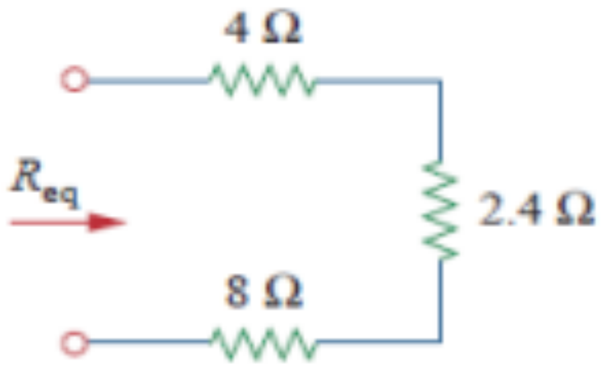
Two resistors 2 and 2 series and parallel with 6

(2+2) // 6 = 2.4 Ω

Three resistances 4, 8 and 2.4 are series

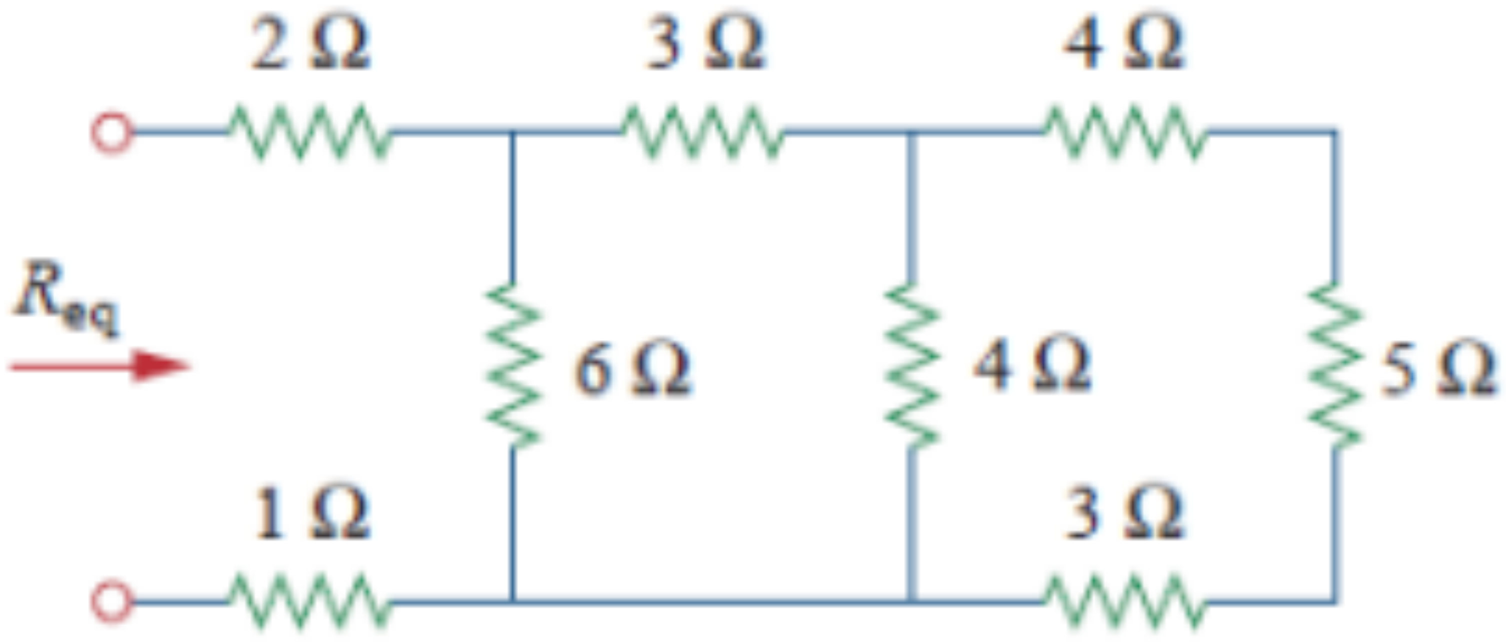
4 + 8 + 2.4 = 14.4 Ω





❑ Example 2.3:

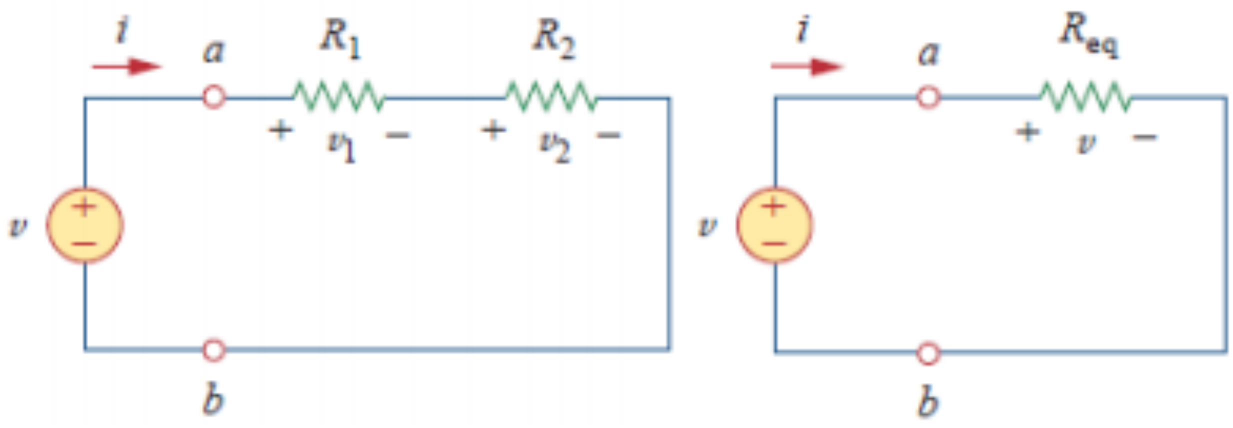
➢ Find Req for the circuit shown in the following figure



Answer: 6 Ω.

2.4 Voltage and current division

- To determine the voltage across each resistor by using voltage divider in figure 4 as the following



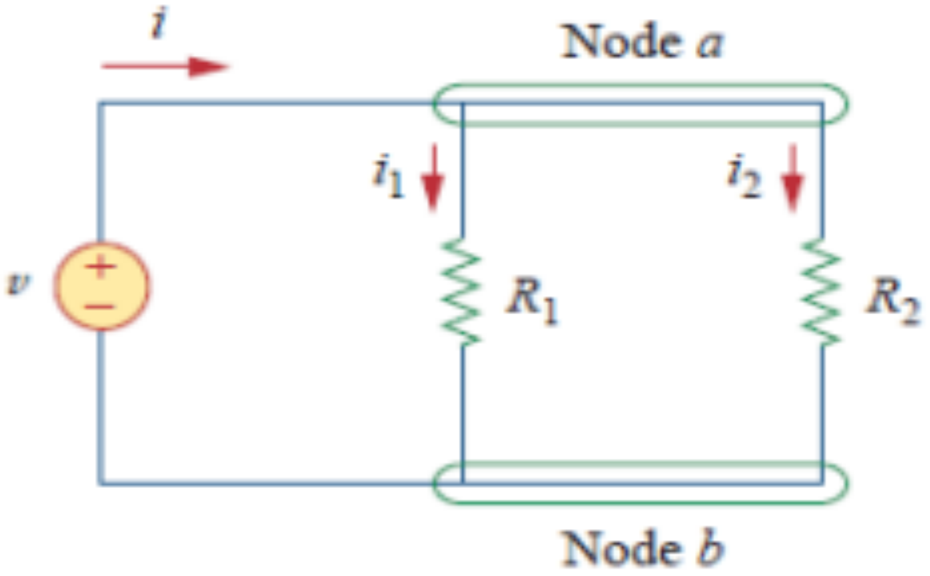
$$v_1 = \frac{R_1}{(R_1+R_2)}v$$

$$v_2 = \frac{R_2}{(R_1+R_2)}v$$

$$v_i = \frac{R_i}{(R_1+R_2+\cdots+R_N)}v$$

❑ 2.4 Voltage and current division

- To determine the current through each resistor by using current divider in figure 5 as the following



$$v = iR_{eq} = \frac{iR_1R_2}{(R_1 + R_2)} = i_1R_1 = i_2R_2$$

$$i_1 = \frac{R_2}{(R_1 + R_2)}i$$

$$i_2 = \frac{R_1}{(R_1 + R_2)}i$$

Nodal Analysis

❑ Nodal analysis

- Nodal analysis provides a general procedure for analyzing circuits using node voltages as the circuit variables.
- Choosing node voltages instead of element voltages as circuit variables is convenient and reduces the number of equations one must solve simultaneously. To simplify matters, we shall assume in this section that circuits do not contain voltage sources.
- Circuits that contain voltage sources will be analyzed in the next section. In nodal analysis, we are interested in finding the node voltages.

❑ Nodal analysis

➢ Given a circuit with n nodes without voltage sources, the nodal analysis of the circuit involves taking the following three steps.

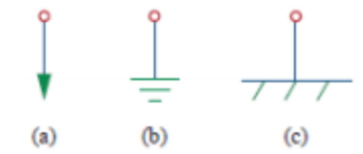


Fig. 6: Symbols of references node.

➢ Select a node as the reference node as shown in figure 6. Assign voltages $v1$, $v2$, … , vn to the remaining (n-1) nodes. The voltages are referenced with respect to the reference node.

➢ Apply KCL to each of the (n-1) non-reference nodes. Use Ohm's law to express the branch currents in terms of node voltages.

➢ Solve the resulting simultaneous equations to obtain the unknown node voltages.

❑ 1. Nodal analysis

- As shown in figure 7, Current flows from a higher potential to a lower potential in a resistor.

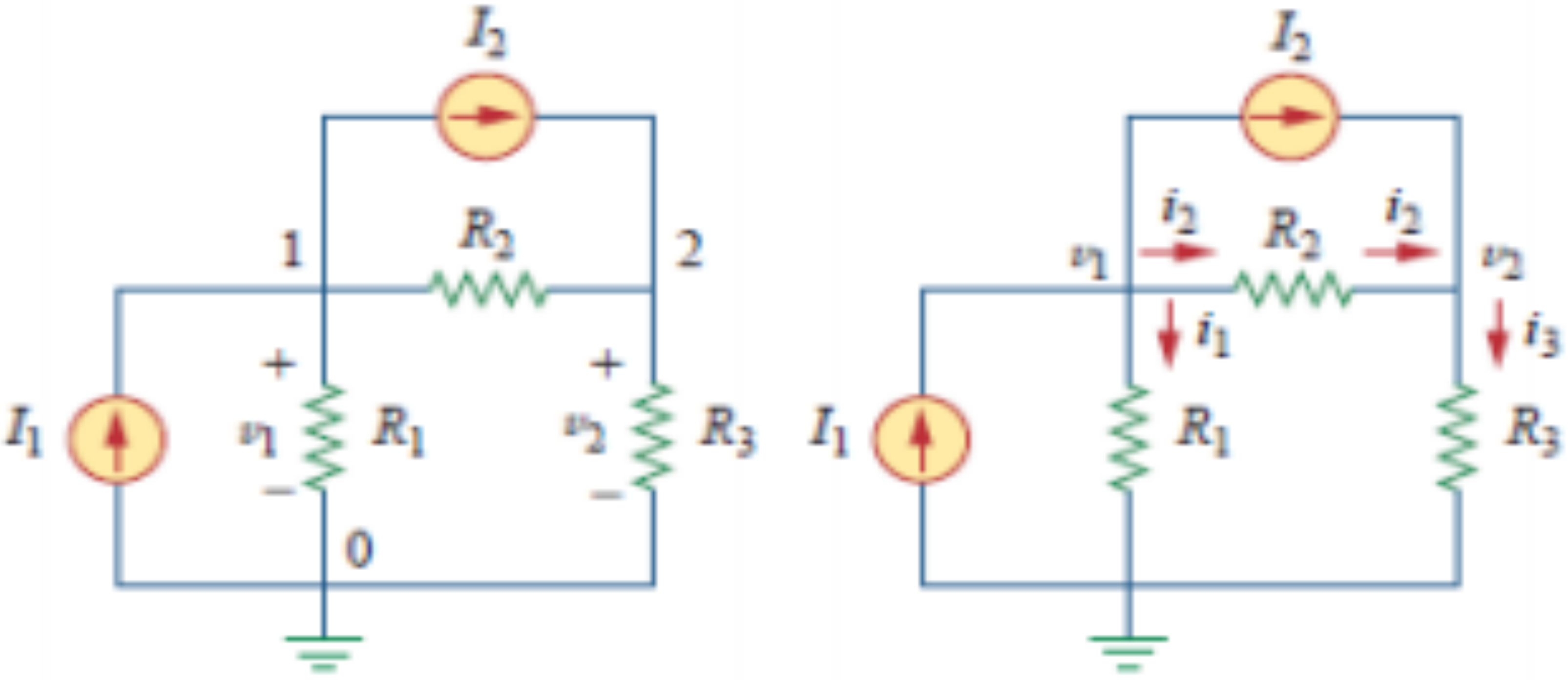
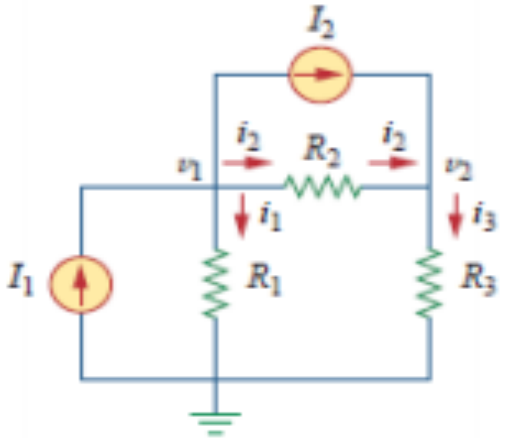


Fig. 7: Typical circuit for nodal analysis.

❑ 1. Nodal analysis

- As shown in figure 7, Current flows from a higher potential to a lower potential in a resistor.

$$i = \frac{v_{higher} - v_{lower}}{R}$$



$$i_1 = \frac{v_1 - 0}{R_1} = G_1 v_1$$

$$i_2 = \frac{v_1 - v_2}{R_2} = G_2(v_1 - v_2)$$

$$i_3 = \frac{v_2 - 0}{R_3} = G_3 v_2$$

node 1:

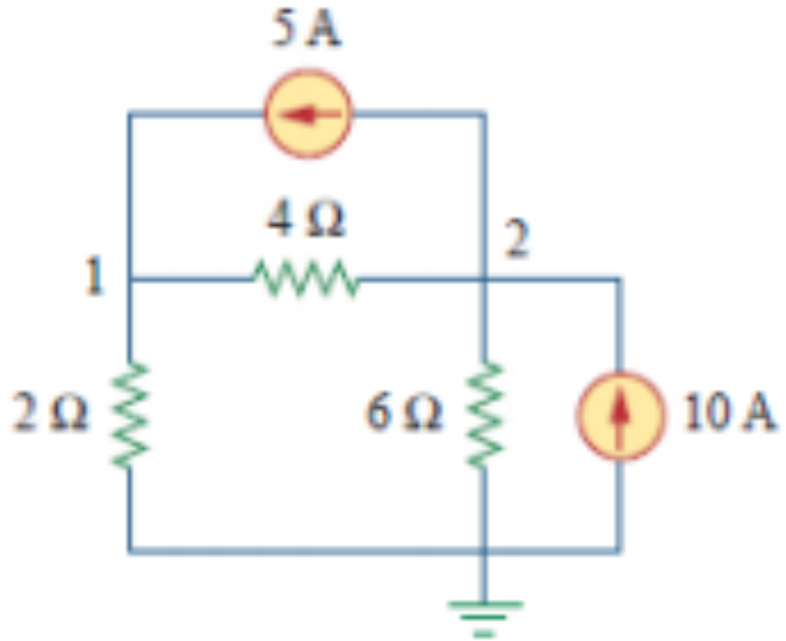
$$I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2}$$

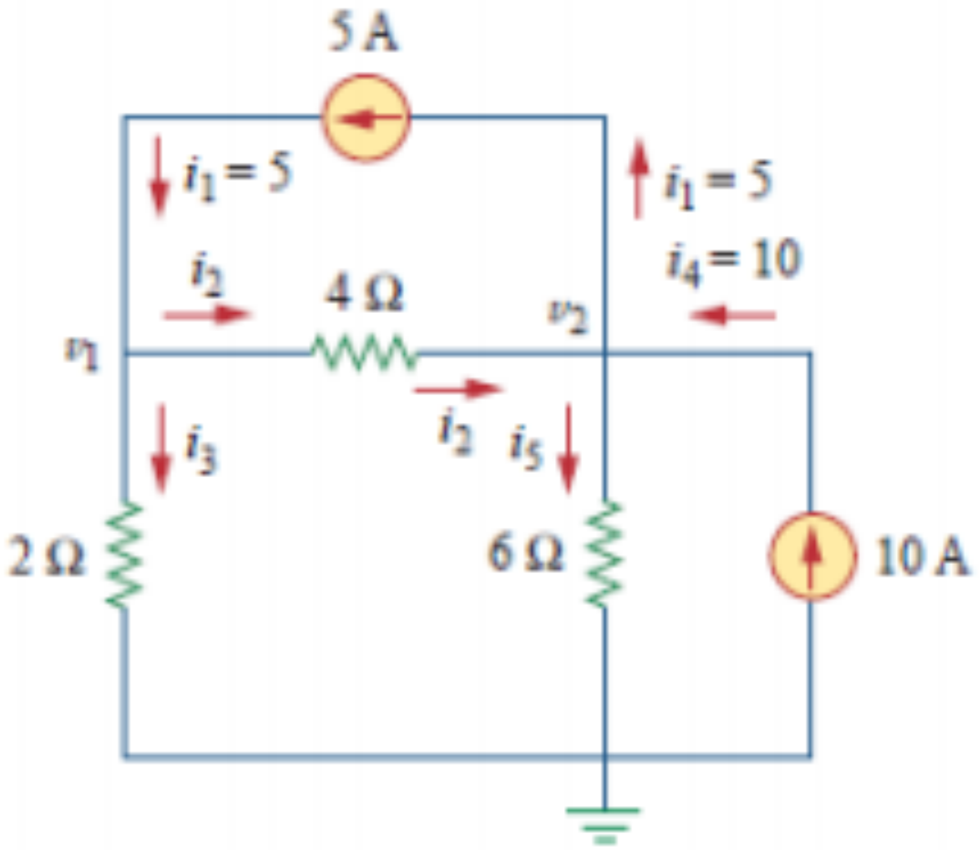
node 2:

$$\frac{v_2}{R_3} = I_2 + \frac{v_1 - v_2}{R_2}$$

❑ Example 1:

➢ Find $v1$and $v2$ using nodal analysis





❑ Example 1:

Answer: At node 1, applying KCL and Ohm's law gives

node 1:

$$i_1 = i_2 + i_3$$

$$5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2} \quad (1)$$

node 2:

$$i_2 + i_4 = i_1 + i_5$$

$$\frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2 - 0}{6} \quad (2)$$

from (1) and (2) $v_1 = 13.33V \quad and \; v_2 = 20V.$

❑ super-node

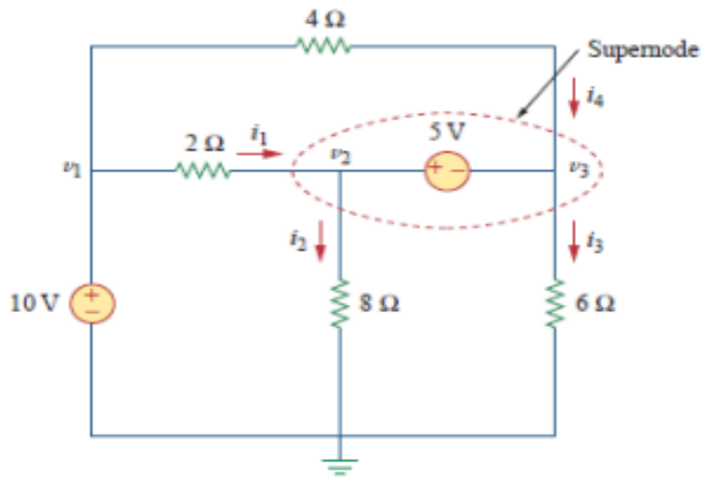
- Nodal analysis with voltage source is called super-node (A super-node is formed by enclosing a (dependent or independent) voltage source connected between two non-reference nodes and any elements connected in parallel with it. and considers as special case.

❑ CASE 1

- If a voltage source is connected between the reference node and a non-reference node, we simply set the voltage at the non-reference node equal to the voltage of the voltage source. In figure 8., for example,

$$v1 = 10V$$

- Thus, our analysis is somewhat simplified by this knowledge of the voltage at this node.



❑ CASE 2

➢ If the voltage source (dependent or independent) is connected between two non-reference nodes, the two nonreference nodes form a generalized node or super-node; we apply both KCL and KVL to determine the node voltages.

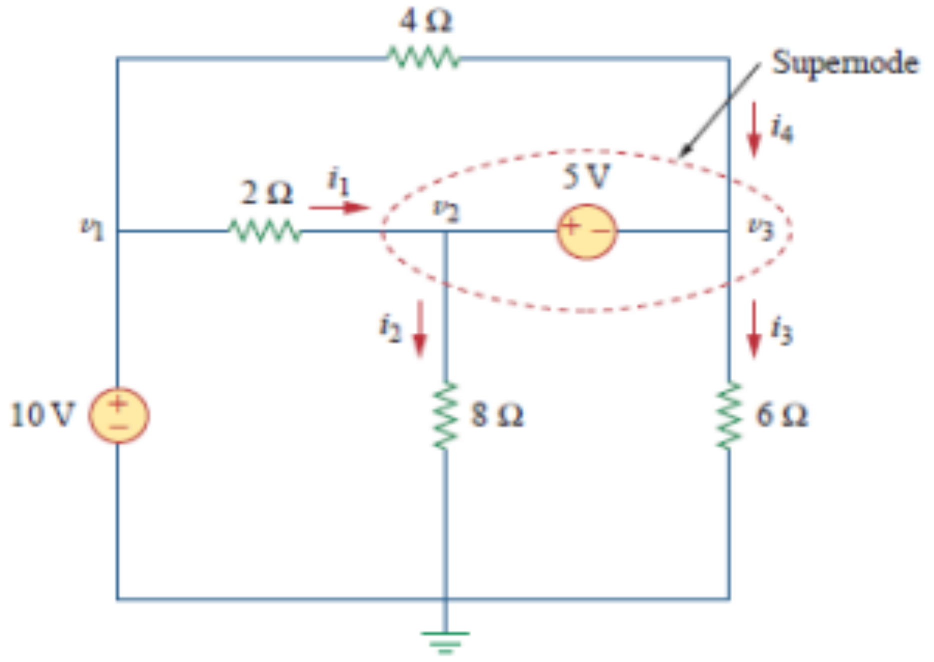
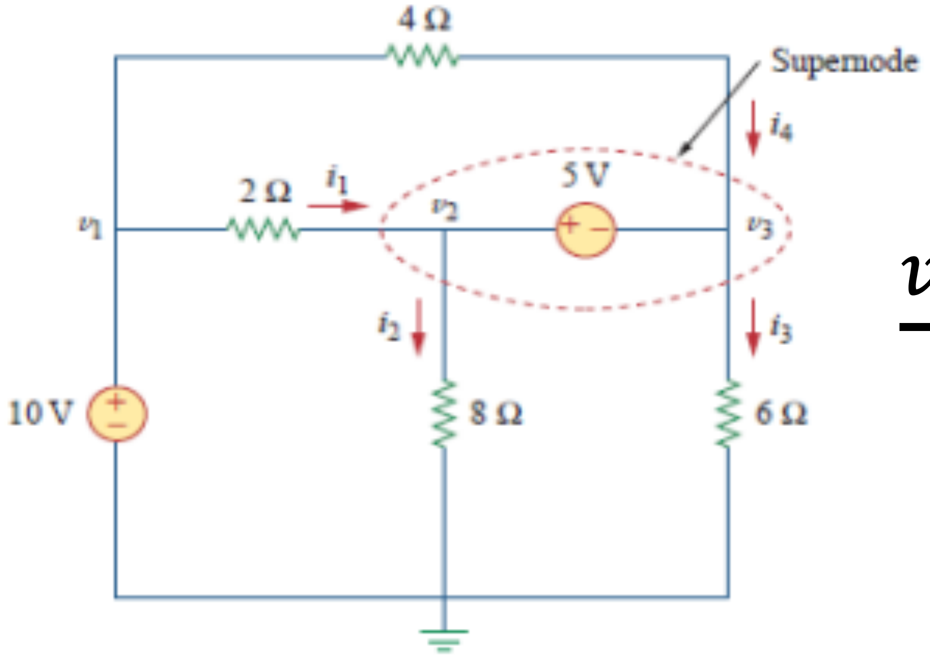


Fig. 8: Nodal analysis with voltage source

CASE 2

- We can solve the following three equation to find all three voltages.



$$i_1 + i_4 = i_2 + i_3$$

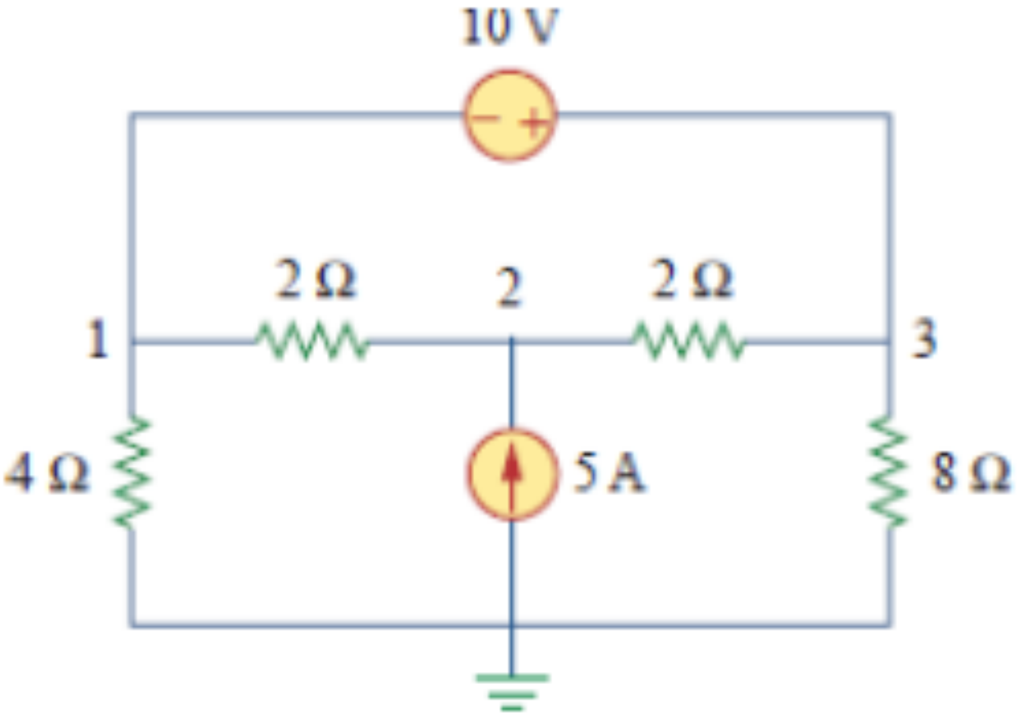
$$\frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} = \frac{v_2 - 0}{8} + \frac{v_3 - 0}{6} \quad (1)$$

$$v_2 - v_3 = 5 \quad (2)$$

$$v_1 = 10 \quad (3)$$

❑ Example

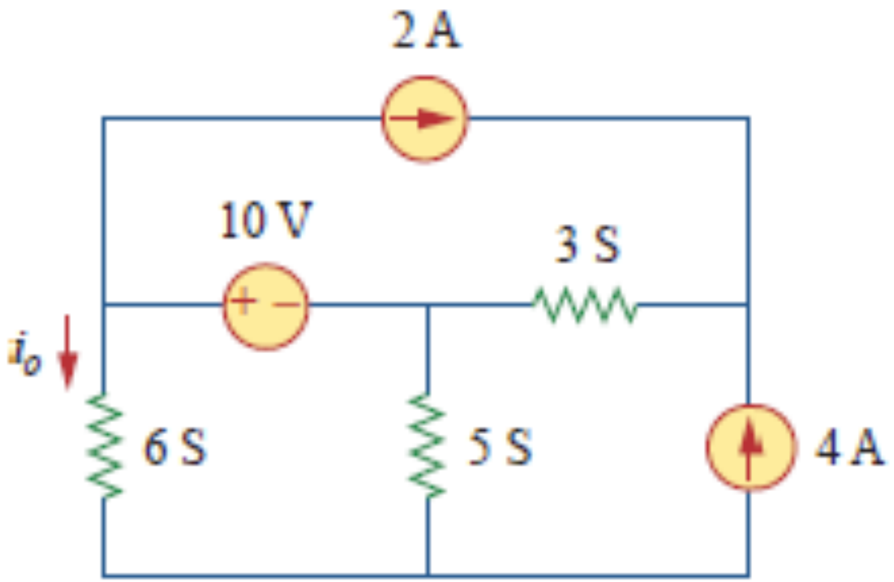
- For the circuit in the following figures, find all node voltages by using nodal analysis



(a)

❑ Example

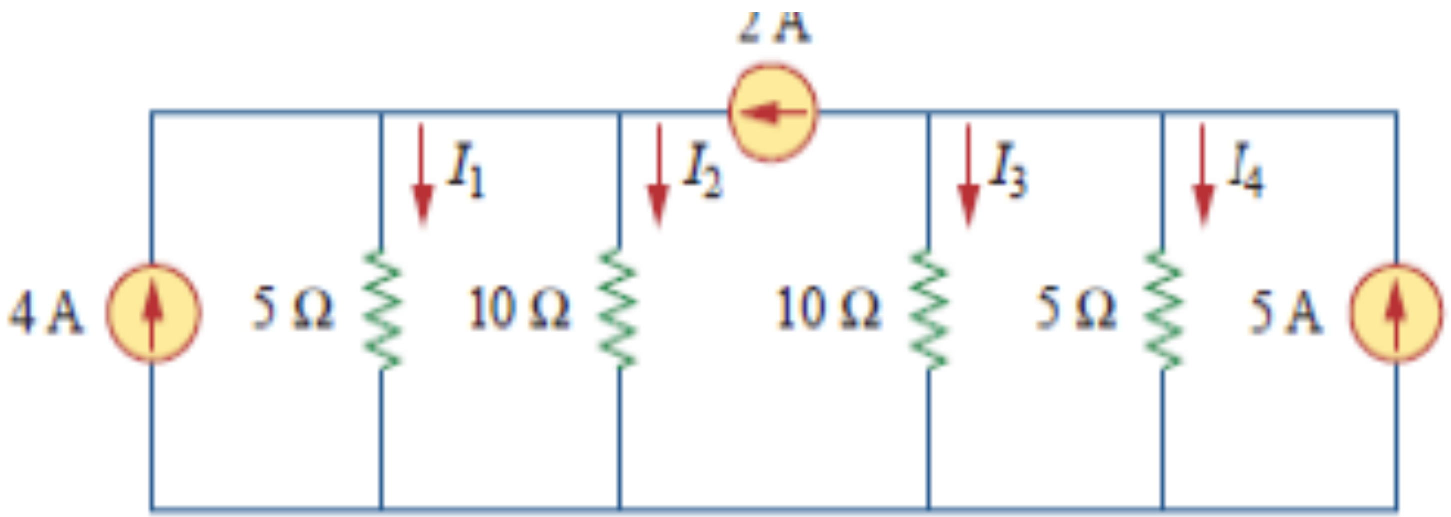
➢ For the circuit in the following figures, find all node voltages by using nodal analysis



(b)

❑ Example

- For the circuit in the following figures, find all node voltages by using nodal analysis



(c)


Thank you